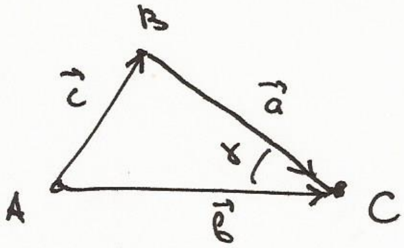
Correction du TD 0 (Ex. 9-16)

Ex. 9



$\vec{c} = \vec{b} - \vec{a} \Rightarrow$

$\vec{c}\cdot\vec{c} = c^2 = (\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a}) =$

$= b^2 + a^2 - 2\vec{a}\cdot\vec{b} = b^2 + a^2 - 2ab\cos\gamma$

D'autre part

$-\vec{c}\cdot\vec{c} + \vec{b}\cdot\vec{b} + \vec{a}\cdot\vec{a} = \vec{b}\cdot\vec{b} + \vec{a}\cdot\vec{a} - (\vec{b}-\vec{a})\cdot(\vec{b}-\vec{a}) =$

$= 2ab\cos\gamma = b_x^2 + b_y^2 + b_z^2 + a_x^2 + a_y^2 + a_z^2 - (b_x - a_x)^2 - (b_y - a_y)^2 - (b_z - a_z)^2$

$= 2(a_x b_x + a_y b_y + a_z b_z).$

Ex. 10 $|\vec{a}\wedge\vec{b}|^2 = (a_y b_z - a_z b_y)^2 + (a_z b_x - a_x b_z)^2 + (a_x b_y - a_y b_x)^2$

$a^2 b^2 \sin^2\varphi = a^2 b^2 - (\vec{a}\cdot\vec{b})^2 = (a_x^2 + a_y^2 + a_z^2)(b_x^2 + b_y^2 + b_z^2) - (a_x b_x + a_y b_y + a_z b_z)^2$

Il reste à vérifier que

$(a_y b_z - a_z b_y)^2 + (a_z b_x - a_x b_z)^2 + (a_x b_y - a_y b_x)^2 =$

$= (a_x^2 + a_y^2 + a_z^2)(b_x^2 + b_y^2 + b_z^2) - (a_x b_x + a_y b_y + a_z b_z)^2$

Ex. 11 $(\vec{a}\wedge\vec{b})\cdot\vec{c} = \begin{pmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{pmatrix} \cdot \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} =$

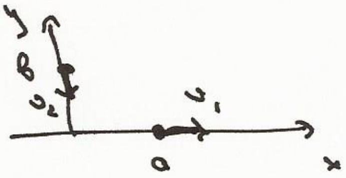
$= (a_y b_z - a_z b_y)c_x + (a_z b_x - a_x b_z)c_y + (a_x b_y - a_y b_x)c_z$

La dernière expression coïncide avec le développement du déterminant $\det\begin{pmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{pmatrix}$ selon la 3ème ligne.

Ex. 12

$$\begin{cases} x = x_A(1-t) + x_B t = -2(1-t) + 3\cdot t = 5t - 2 \\ y = y_A(1-t) + y_B t = 3(1-t) + 4t = t + 3 \\ z = z_A(1-t) + z_B t = 1\cdot(1-t) + 7t = 6t + 1 \\ t \in [0,1] \end{cases}$$

Ex. 13



$$\begin{cases} x_1(t) = a + v_1 t \\ y_1(t) = 0 \end{cases}$$

$$\begin{cases} x_2(t) = 0 \\ y_2(t) = b - v_2 t \end{cases}$$

$$d_{12}^2 = (a + v_1 t)^2 + (b - v_2 t)^2$$

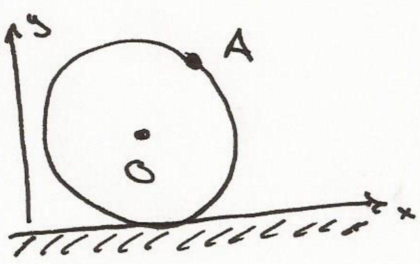
$$\frac{d\left(d_{12}^2\right)}{dt} = 2(a+v_1 t)\cdot v_1 - 2(b - v_2 t)v_2 = 2(av_1 - bv_2) + 2(v_2^2 + v_1^2)t = 0$$

$$\hookrightarrow t = \frac{-av_1 + bv_2}{v_2^2 + v_1^2} \Rightarrow (d_{12}^2)_{min} = a^2 + b^2 - \frac{(bv_2 - av_1)^2}{v_1^2 + v_2^2} =$$

$$= \frac{a^2 v_2^2 + b^2 v_1^2 + 2ab v_1 v_2}{v_1^2 + v_2^2} = \frac{(av_2 + bv_1)^2}{v_1^2 + v_2^2}$$

Par conséquent, $d_{min} = \dfrac{av_2 + bv_1}{\sqrt{v_1^2 + v_2^2}}$

Ex. 14



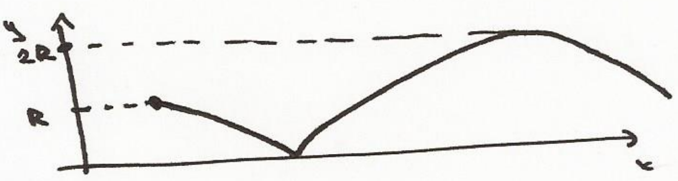
$$\begin{cases} x_O(t) = vt \\ y_O(t) = R \end{cases}$$

roulement sans glissement: $v = \omega R$

$$\begin{cases} x_{A/O}(t) = R\cos\omega t \\ y_{A/O}(t) = -R\sin\omega t \end{cases}$$

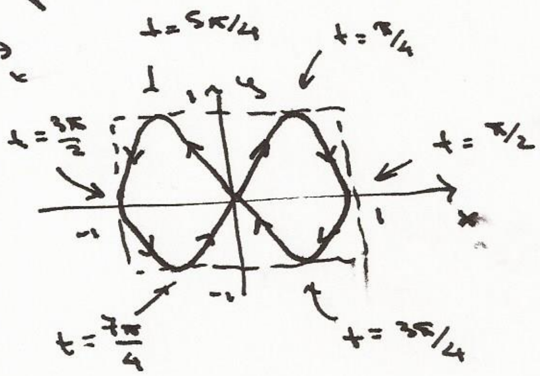
Equations paramétriques:

$$\begin{cases} x(t) = x_O(t) + x_{A/O}(t) = \omega R t + R\cos\omega t \\ y(t) = y_O(t) + y_{A/O}(t) = R - R\sin\omega t \end{cases}$$



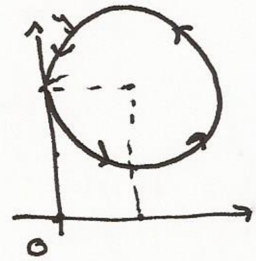
Ex. 15

$$\begin{cases} x(t) = \sin t \\ y(t) = \sin 2t \\ t \in [0, 2\pi] \end{cases}$$

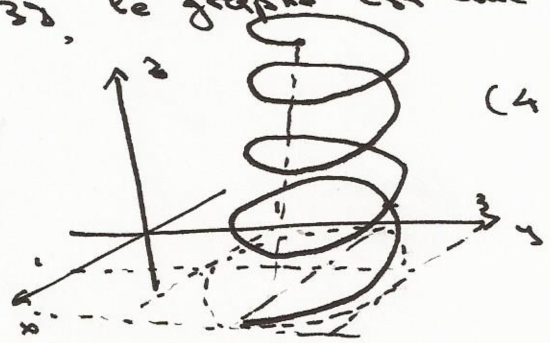


$$\begin{cases} x(t) = 1 + \cos t \\ y(t) = 2 + \sin t \\ z(t) = \dfrac{t}{2\pi} \\ t \in [0, 8\pi] \end{cases}$$

Dans le plan xy nous avons un cercle $(x-1)^2 + (y-2)^2 = 1$ parcouru 4 fois :



En 3D, le graphe est une spirale



(4 tours)

Ex. 161
$$\begin{cases} x(u,v) = -u + v \\ y(u,v) = 2u \\ z(u,v) = -u - v - 1 \\ (u,v) \in \mathbb{R}^2 \end{cases}$$

5 paramètres
$-$ 3 équations

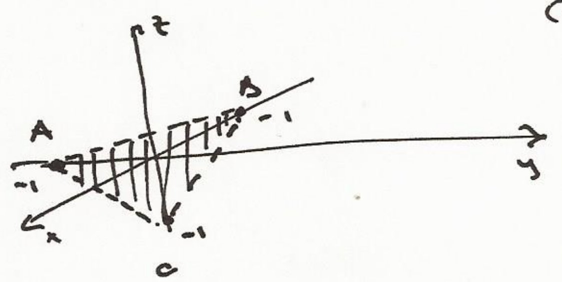
2 paramètres (une surface)

En additionnant x, y, z, on obtient

$x + y + z = -u + v + 2u - u - v - 1 = -1$

$\hookrightarrow$ on obtient le plan $x + y + z + 1 = 0$

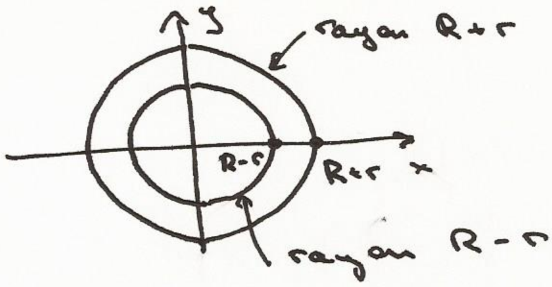
(plan du triangle ABC)



$$\begin{cases} x(u,v) = (R + r\cos v)\cos u \\ y(u,v) = (R + r\cos v)\sin u \\ z(u,v) = r\sin v \\ u \in [0, 2\pi], \quad v \in [0, 2\pi] \end{cases}$$

intersection avec le plan $z = 0 \Rightarrow v = 0, \pi$

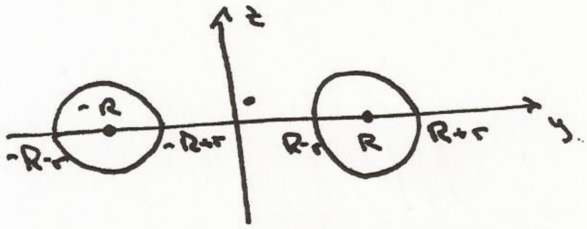
$$\Rightarrow \begin{cases} x = (R - r)\cos u \\ y = (R - r)\sin u \\ u \in [0, 2\pi] \end{cases} \qquad \begin{cases} x = (R + r)\cos u \\ y = (R + r)\sin u \\ u \in [0, 2\pi] \end{cases} \qquad \text{(2 cercles)}$$



intersection avec le plan yz : $x=0 \Rightarrow u = \frac{\pi}{2}, \frac{3\pi}{2}$

$$\begin{cases} y(v) = R + r\cos v \\ z(v) = r\sin v \\ v \in [0, 2\pi] \end{cases} \qquad \begin{cases} y(v) = -(R + r\cos v) \\ z(v) = r\sin v \\ v \in [0, 2\pi] \end{cases}$$

(2 cercles)



+ intersection similaire avec le plan xz

En 3D, on obtient un tore :